Reshaping Inflation Expectations at the Zero Lower Bound

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Abstract

We investigate the anchoring and stabilization powers of forward-guidance on macroeconomic dynamics in so-called liquidity traps. The research proceeds within the baseline New Keynesian model in which non-linearities give rise to multiple equilibria and deflationary recession paths. Considering the major puzzle raised by homogeneous and rational expectations in this model, we introduce heterogeneous expectations using a social learning algorithm (Arifovic et al., 2012). The resulting framework displays *local* stability and fits better US data and especially persistence at the Zero Lower Bound (ZLB) than a switching regime rational expectation model(Guerrieri and Iacoviello, 2015a). Nonetheless, our model displays a strong expectation feedback loop: when agents' assess solutions signaled and expectations perform better than the central bank's signal, these expectations could loose their anchorage at the targeted steady state. In this model, instability is driven by a expectation miscoordination at the ZLB rather than strong exogenous shocks. We find that if the central bank signals the rational expectation solution, the stability of the economy is significantly improved and expectations stay anchored at the cost of an higher forecast dispersion.

Keywords: learning, forward guidance, bounded rationality, heterogeneous expectations, non-linear dynamics
JEL Classification: **C69, D83, E03, E31, E52, E58, E61**

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1 Introduction

The recent macro-economic performances in the turmoils of the Great Recession have renewed the interest in the concept of liquidity trap (Keynes, 1936), i.e. the coexistence of near-zero inflation with poor economic performances. In such a context, the zero-lower bound (ZLB) constraints the nominal interest rates, which are the primary stabilizing instrument of central banks. Additionally, the increase in sovereign risk premia and the rising debt burden have rapidly constrained the fiscal instrument, especially in the Euro area. Meanwhile, inflation expectations have declined steadily.Therefore, central banks have deployed a large range of alternative and innovative tools and policy measures. Aside the so-called Quantitative Easing, central banks have reinforced their communication policy to act on the expectations of the private sector. In particular, at the ZLB, communication has a bigger role than in a rather stable environment because central bank projections are not just a purely informational tool, but also acts as a strategic instrument to influence inflation expectations and, hence, the real interest rate once the nominal interest rate is set to zero (Charemza and Ladley, 2016). These communication measures in "crisis time" are referred to as forward-guidance which include pure forecasts – i.e. Delphic forward guidance – or extensive communication about future rates, broader transparency and public self-imposed rules – i.e. Odysean forward guidance.

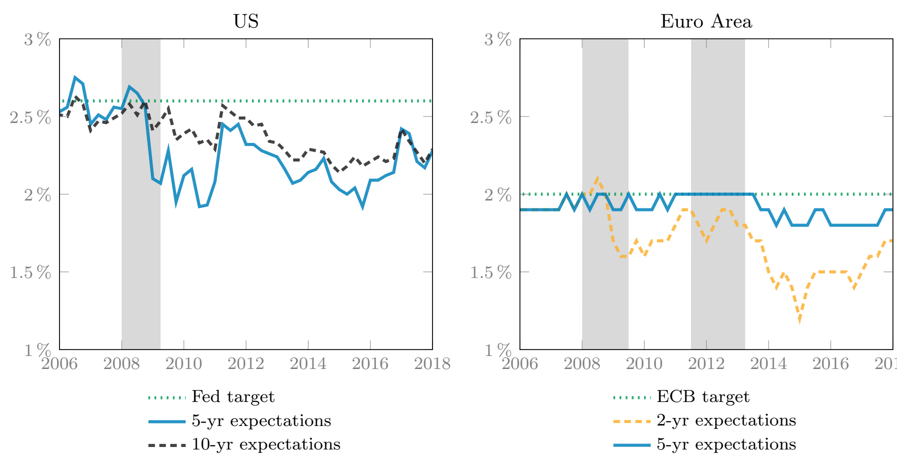

Figure 1: Inflation expectations in the U.S and Euro-zone during ZLB periods

However, several conditions need to be present for forward guidance to be effective (Blinder et al., 2008). The economy should not be stationary, but displays instead indeterminacy regions or multiple steady states. Moreover, either agents should face some form of bounded rationality in their expectations (Sargent and Others, 1993; **?**) or some information asymmetry w.r.t. the central bank. As a corollary, agents should be able to learn and and revise their expectations from the central bank communication.

Hence, this paper develops a framework that displays those conditions. More precisely, we use a non-linear New Keynesian (NK) dynamic stochastic general equilibrium (DSGE) model in which non-linearities create multiple equilibria and agents are boundedly rational. They form heterogeneous expectations using social learning modeled with an evolutionary algorithm [1]

The question of coordination between heterogeneous agents is also particularly relevant for forward guidance.There is extensive experimental evidence (Hommes, 2011) and empirical works on forecasters (Mankiw et al., 2003; Landier and Thesmar, 2017; Carroll, 2003) or consumers (Cavallo et al., 2014) showing that heterogeneity in computation capacities and priors belief affect in a non canonical way agents' expectations. However, the rational and representative agent paradigm rules out any coordination issue (Kaldor, 1972; Kirman, 2016), and the potential for coordination failures (Guesnerie, 2010). However, those coordination failures may be the origin of significant macroeconomic volatility.

In this research, we choose to rely on evolutionary learning to model expectations in the macroeconomic framework that we consider. Our motivations come both from the theoretically appealing features of these models (Lux and Schornstein, 2005) and their ability to match experimental findings (Arifovic and Ledyard, 2012) and some empirical surveys. Three features are especially well-suited for the analysis of forward-guidance in an expectation-driven liquidity trap. First, social learning allows us to explicitly account for some heterogeneity in expectations, and to tackle the related question of their coordination on the objectives of the central bank. Second, social learning is able to feature announcements and news into agents' expectation formation process, and therefore allows us to integrate a forward-looking component in a learning mode(Arifovic and Ledyard, 2012). Additionally, non-linearities, multiple equilibria, heterogeneity of expectations, and expectation driven dynamics complicate the learning and coordination process of agents that populate our model. The resulting framework works along the line of a so-called "complex systemâĂİ, which requires for the agents to be able to adapt to an ever-changing environment, in which their own actions and expectations impact upon the resulting macroeconomic dynamics, which in turn feedback their expectations. An evolutionary learning process allows us to model adaptation in such a complex environment in a parsimonious way.

With this framework at hand, we seek to answer the following questions: What are the stability properties under evolutionary learning of the different steady states? Is social learning a suitable setup to explain ZLB dynamic and low inflation persistence? Can central bank forward guidance policy steer expectations away from the unintended stagnation equilibrium towards the targeted steady state? Or, can forward guidance help keep expectations anchored at the target and prevent a fall in a stagnation trap in case of strong adverse shocks? If yes, under which conditions?

[1] see Arifovic (2000), Arifovic et al. (2012),and Arifovic et al. (2017)

Some of these questions have been tackled under rational expectations (see Guerrieri and Iacoviello (2015a) and Borağan Aruoba et al. (2017)) and under homogeneous expectations formed by adaptive learning (Bullard et al., 2002). The empirical literature also provides some evidence. We summarize this literature in a separate section below. The main innovative feature of this paper is to introduce heterogeneous expectations within a baseline NK model with non-linerarities and multiple steady states. This allows us to explicitly address the question of agents' coordination. We model forward guidance in the form of providing the agents with the knowledge about the future interest rates or the central bank's corresponding inflation and/or output forecasts. We then ask how can forward guidance act upon the heterogeneity of non aggregate expectations to achieve coordination between agents on the targeted state? Again, how do the effects of forward guidance depend on the monetary policy design or the learning parameters of the model?

We show in this paper that our model is able to reproduce the properties of the US macro-economic dynamics between 1947-2017. The model is stable under learning and able to transition between the ZLB constrain state and non constrain state without ad-hoc switching mechanism. A forward guidance in the form a MSV signal stabilizes efficiently the model and avoid endogenous switching. Yet, when credibility of the CB's is assessed, forward guidance hindered the model stability.

This paper proceeds as follows. Section 2 discusses the related literature. In Section 3, we develop the theoretical framework, the stability properties of the model are analyzed in Section 5 and the estimation strategy in Section4. Section 6 presents our results and Section 7 concludes.

2 Literature review

The academic literature provides fairly recent results on the effects of forward guidance at the ZLB. On the empirical side, the literature provides supportive conclusions (Del Negro et al., 2012; Hubert, 2015; Andrade et al., 2015; Campbell et al., 2016). Few laboratory experiments provide more mixed evidence, see Kryvtsov and Petersen (2016); Mokhtarzadeh and Petersen (2016); Arifovic and Petersen (2015); Amano et al. (2011).

From a theoritical viewpoint, forward guidance within rational expectation models has given rise to the so-called "forward guidance puzzleâĂİ: standard DSGE models predict explosive inflation and output dynamic when the short-term interest rate hits the ZLB for an extended period of time (Carlstrom et al., 2012). However, in the Eurozone and the U.S, interest rates have been pegged to zero for much longer horizons without substantial changes in the price levels. Thus, showing that DSGE models fails at explaining the behavior of the economy at the ZLB, Del Negro et al. (2012) point out that this puzzle results from the core ingredients and the calibration of those DSGE models that are widely used both in academia and in policy making institutions. A first innovation upon the standard way that DSGE models are used is to use rational expectation models in their non-linear form. Within non-linear models, multiple equilibria arise, and some of these equilibria correspond to a liquidity trap state (McCallum, 2003). However, this does not solve the puzzle: in this class of models, a liquidity trap gives rise to unstable dynamics (Fernández-Villaverde et al., 2015; Ascari and Rossi, 2009), characterized by excess macroeconomic volatility or microfondation puzzle (Guerrieri and Iacoviello (2015a) and Boragan Aruoba et al. (2017)) under rational expectations (Benhabib et al., 2001a,b), and deflationary spirals under adaptive learning (Evans et al., 2008).

Nonetheless, heterogeneous expectations (Farhi and Werning, 2017) and behavioral approach (Goy et al., 2016) alongside space learning (Gabaix, 2016) in DSGE models have lead to some improvement in the forward guidance puzzle issue.

Several papers rely on non-linerarities to account for the recent state of affairs. use an extension of a non-linear NK model enabled by the Rotemberg pricing rule that has an additional steady state characterized by stagnation trap, see Benhabib et al. (2001a,b), Evans et al. (2008), Evans et al. (2016), Arifovic et al. (2017), Hommes et al. (2015) and Lansing (2017). This steady state is locally determinate under rational expectations, and locally stable under learning, like the targeted steady state. Those models therefore provide a unified framework to account for both periods of macroeconomic stability and of stagnation below the targeted levels of economic activity and inflation.

In this class of models, agents' expectations play a key role: only negative shocks on expectations can push inflation and output away enough from their targeted levels to engage them on a stagnation equilibrium path. Put differently, this class of models offers a reading of the current economic situation as a coordination of agents' expectations on a locally stable equilibrium path which is socially sub-optimal, but co-exists next to the socially desirable, targeted equilibrium path. Evans et al. (2008), Evans et al. (2016) and Hommes et al. (2015) investigate the role of fiscal and monetary policy mix in this context, but do not tackle the effects of forward guidance and central bank communication.

This paper does not use a non-linear model, but instead imposes the non-linearities in the log-linearized form derived from the widely-used sticky prices setup à la Calvo (1983). This feature, which admittedly decreases precision of the dynamics far enough from the steady states, enables us to stress-out the multiplicity of steady states, with a normal times targeted state, a deflationary state and a stagnation state, while keeping the model simple enough to introduce heterogeneous expectations.

3 A behavioural New Keynesian model

3.1 The model

Our model builds on the standard baseline three-equation NK model developed by, *inter alia*, Woodford (2011).

The model consists in the following Euler equation with $\widehat{y}$ the output gap, $\widehat{\pi}$

the inflation rate deviation from the target, $\widehat{i}$ the nominal interest rate deviation set by the central bank, $\widehat{g}$ and $\widehat{u}$ two exogenous variations of the demand and supply, (due to a so-called real shocks) and σ the intertemporal elasticity of substitution of consumption (based on a CRRA utility function). $\overline{r} = \pi^* \beta^{-1}$ is the natural level of interest rate and π^* the quarterly targeted level of inflation.

$$\widehat{y_t} = E_t^i \widehat{y}_{t+1} - \sigma^{-1}(\widehat{i}_t - E_t^i \widehat{\pi}_{t+1}) + \widehat{g_t} \tag{1}$$

The supply side is summarized by the forward looking New Keynesian Philips Curve where $0 < \beta < 1$ represents the discount factor and $\kappa > 0$ a composite parameter capturing the slope of the Phillips curve. κ could be decomposed in this manner $\kappa = (\delta + \sigma)(1-\theta)\frac{1-\beta\theta}{\theta}$ with θ the Calvo update setting and γ the labor elasticity.

$$\widehat{\pi}_t = \kappa \widehat{y_t} + \beta E_t^i \widehat{\pi}_{t+1} + \widehat{u}_t \tag{2}$$

The real shocks follows an exogenous auto-regressive disturbance with ε_t^g and ε_t^u random draws normal centered with the respective standard deviation sd^g and sd^u.

$$\widehat{g}_t = \rho^g \widehat{g}_{t-1} + \varepsilon_t^g \tag{3}$$

$$\widehat{u}_t = \rho^u \widehat{u}_{t-1} + \varepsilon_t^u \tag{4}$$

With $0 < \rho^u < 1$ and $0 < \rho^g < 1$ the persistence of both shocks. We use the standard forward looking Taylor rule where ϕ^π and ϕ^y are the respective reaction coefficients to output and inflation gaps.

$$\widehat{i}_t = \phi^\pi E_t^i \widehat{\pi}_{t+1} + \phi^y E_t^i \widehat{y}_{t+1} \tag{5}$$

Adding (5) into (1) and the result into (2) we can write the model's policy function into this reduced form:

$$z_t = \alpha + B E_t^i z_{t+1} + \chi^g \widehat{g}_t + \chi^u \widehat{u}_t \tag{6}$$

with $z_t = \begin{bmatrix} \widehat{y}_t \\ \widehat{\pi}_t \end{bmatrix}$,$\alpha = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $B = \begin{bmatrix} 1-\sigma^{-1}\phi^y & \sigma^{-1}(1-\phi^\pi) \\ \kappa(1-\sigma^{-1}\phi^y) & \beta + \sigma^{-1}(1-\phi^\pi)\kappa \end{bmatrix}$, $\chi^g = \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$ and $\chi^u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

We are interested in states far from the targeted steady state, which implies the use of non-linearities. The first one is most common. Following e.g. Nakov (2008), we had a ZLB to the monetary policy rule:

$$i_t = max\{-\overline{r}; \phi^\pi E_t^i \widehat{\pi}_{t+1} + \phi^y E_t^i \widehat{y}_{t+1}\} \tag{7}$$

In addition, following Evans et al. (2016), we consider the existence of an inflation and output lower bounds respectively named $\underline{y}$ and $\underline{\pi}$. This feature enables us to stabilize the output in a lower bound after an unstable depressive episode in the indeterminate region of the state space. These bounds can be explained by an extreme downward rigidity of wages near the subsistence level or the existence of inter-temporal arbitrages. Thus, we end up with another steady state

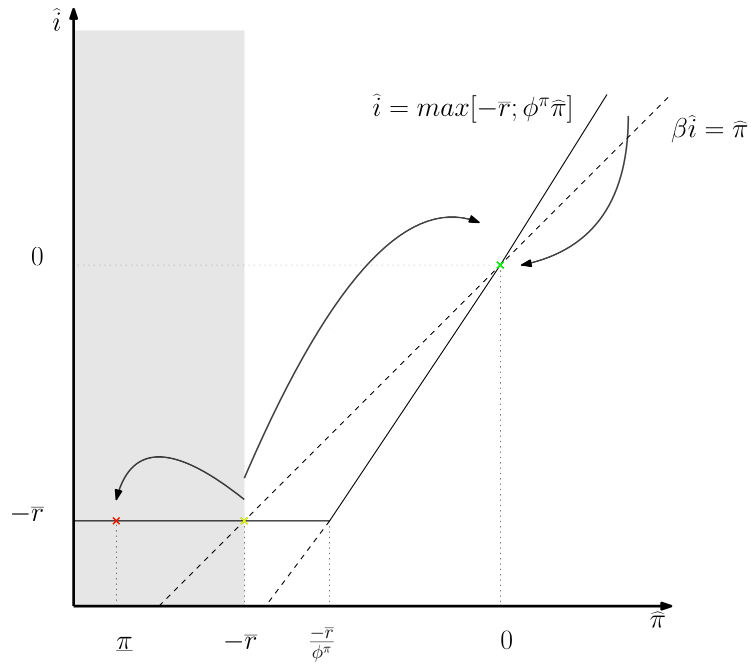

Figure 2: Existence of multiple steady state with ZLB and inflation lower bond (grey zone is indeterminate under rational expectation

at the inflation lower bound that we call the stagnation state, following Evans et al. (2016).

Adding the ZLB constrained generates a deflation steady state when deflation rate matches the discount factor. At this moment, agents are indiferent between risk-free bonds and consumption. Evidence for the existence of this lower bound are discussed theoretically in Benhabib et al. (2001b) and empirically in IMF. As shown by these works, a non linear Taylor rule implies the existence of multiple steady states : the social optimum or targeted steady states ($\hat{y} = 0, \hat{\pi} = 0$) and others, including low inflation steady state ($\hat{y}^{zlb}, \hat{\pi}^{zlb}$) and ($\underline{\pi}, \underline{y}$). This multiplicity is illustrated in a one-dimensional system with only inflation and interest rate dynamics in figure 2: provide that $\underline{\pi} < \hat{\pi}^{zlb} \leq 0$, two equilibria exist when the Fisher equation intersect the non-linear Taylor rule, and the lower bound is a way to stabilize the unstable dynamics of the model arising for inflation values for which the Taylor condition does not hold.

We can then write the log approximated Fisher equation as follows:

$$\hat{i} = \beta\hat{\pi} \tag{8}$$

at the Targeted steady state where no deviation occurred :

$$\hat{i} = \beta\hat{\pi} = 0 \tag{9}$$

And at the ZLB, we can derive an equilibrium such that:

$$-\overline{r} = \beta\widehat{\pi} \Leftrightarrow \widehat{\pi}^{zlb} = \beta(1 - \pi^* \beta^{-1}) \tag{10}$$

3.2 Solving the model under RE

We now solve the model under REE and describe the dynamics under Social Learning. Expressing the model in reduced form is challenged by the non-linearities and we need to express it in three pieces around each of the three steady states: around the targeted steady state, when the ZLB is binding and when lower bounds are binding. Using the method of the undetermined coefficients, we express expectations in the form:

$$z_t^e = a + c\rho^g \widehat{g}_t + d\rho^u \widehat{u}_t \tag{11}$$

Under rational expectations, we can write in this fashion :

$$Ez_{t+1}^e = a + cg_{t+1} + du_{t+1} \Leftrightarrow Ez_{t+1}^e = a + c(\rho^g \widehat{g}_t + \varepsilon_t^g) + d(\rho^u \widehat{u}_t + \varepsilon_t^u) \tag{12}$$

and because ε_t^g and ε_t^u are normalized to centered values, we deduce $F(\varepsilon_t^g) = 0$ and $F(\varepsilon_t^u) = 0$ and we can write the following expectation process:

$$Ez_{t+1}^e = a + c\rho^g \widehat{g}_t + d\rho^u \widehat{u}_t \tag{13}$$

Plugging (35) into (12) we have:

$$z_t = \alpha + B[a + c\rho^g g_t + d\rho^u u_t] + \chi^g g_t + \chi^u u_t \Leftrightarrow z_t = \alpha + Ba + g_t(Bc\rho^g + \chi^g) + u_t(Bd\rho^u + \chi^u) \tag{14}$$

Thus, the rational expectation solution is the following system :

$$\begin{cases} \overline{a} = (I - B)^{-1}\alpha \\ \overline{c} = (I - B\rho^g)^{-1}\chi^g \\ \overline{d} = (I - B\rho^u)^{-1}\chi^u \end{cases} \tag{15}$$

When the ZLB is not binding if $(\phi^y(\overline{a}_1 + \overline{c}_1\rho^g\widehat{g}_t + \overline{d}_1\rho^u\widehat{u}_t) + \phi^\pi(\overline{a}_2 + \overline{c}_2\rho^g\widehat{g}_t + \overline{d}_2\rho^u\widehat{u}_t) > -\overline{r})$ our results are conformed to previous established ones. Plugging the Taylor rule into the Euler equation and the result into the NKPC, we deduce that the model can be written in a reduced form:

$$z_t = \alpha + BE_t z_{t+1} + \chi^g g_t + \chi^u u_t \tag{16}$$

with $z_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}$, $\alpha = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 - \sigma^{-1}\phi^y & \sigma^{-1}(1 - \phi^\pi) \\ \kappa(1 - \sigma^{-1}\phi^y) & \beta + \sigma^{-1}(1 - \phi^\pi)\kappa \end{bmatrix}$, $\chi^g = \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$ and $\chi^u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ Using those matrices, thee solution satisfy equilibrium and uniqueness conditions such as: $\phi^y < \sigma(1 + \beta^{-1})$ and $0 < \kappa(\sigma^\pi - 1) + (1 + \beta)\sigma^y < 2\sigma(1 + \beta)$.

In the same manner, when the ZLB is binding $(\phi^y(\overline{a}_1 + \overline{c}_1\rho^g\widehat{g}_t + \overline{d}_1\rho^u\widehat{u}_t) + \phi^\pi(\overline{a}_2 + \overline{c}_2\rho^g + \overline{d}_2\rho^u\widehat{u}_t) < -\overline{r})$ but the Taylor's condition still holds $(\pi^e_{t+1} > -\overline{r})$, we substitute the monetary policy equation into both equations and end up with an RE equilibrium solution:

$$z_t = \alpha^{zlb} + B^{zlb} z^e_{t+1} + \chi^g \widehat{g}_t + \chi^u \widehat{u}_t \tag{17}$$

with $z_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}$, $\alpha^{zlb} = \begin{bmatrix} \sigma^{-1}\overline{r} \\ \kappa\sigma^{-1}\overline{r} \end{bmatrix}$ and $B^{zlb} = \begin{bmatrix} 1 & \sigma^{-1} \\ \kappa & \beta + \sigma^{-1}\kappa \end{bmatrix}$.

Yet, this RE solution is explosive and does not satisfy Blanchard, Olivier; Kahn (1980) conditions because on the eigenvalues on the B^{zlb} matrix do not lie within the unit circle. This is consistent with previous analysis and our upcoming numerical result that shows that the model is unstable at this equilibrium.

When Taylor's conditions do not hold (i.e. when $\overline{a}_2 + \overline{c}_2\rho^g\widehat{g}_t + \overline{d}_2\rho^u\widehat{u}_t < -\overline{r}$), the model falls into indeterminacy and the lower bounds on output and inflation eventually bind. The rational expectation result satisfies:

$$y_t = \underline{y} + \rho^g g_t \tag{18}$$

$$\pi_t = \underline{\pi} + \kappa\rho^g\widehat{g}_t + +\rho^u\widehat{u}_t \tag{19}$$

in a reduced form:

$$\underline{z_t} = \underline{\alpha} + \underline{B} z^e_{t+1} + \chi^g \widehat{g}_t + \chi^u \widehat{u}_t \tag{20}$$

with $\underline{z_t} = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}$, $\underline{\alpha} = \begin{bmatrix} \underline{y} \\ \underline{\pi} \end{bmatrix}$ and $\underline{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

3.3 Expectations under social learning

We now introduce a process of expectation formation based on an evolutionary learning algorithm along the lines of Arifovic et al. (2012) and Arifovic et al. (2017). This algorithm models heterogeneous expectations evolving through a genetic algorithm (Holland, 1975; Dawid, 1999; Black, 1995), which evolves through stochastic and interaction processes. Social learning introduces two mechanisms that homogeneous rational expectation models rule out: coordination issue and bounded rationality. Agents explore the solution space $(\widehat{\pi}^e, \widehat{y}^e)$ thanks to stochastic innovations and an imitation process. In each period, agents evaluate the performance of their forecasts compared to the realizations of the variables and revise them to increase their accuracy for the next period.

There are N agents, indexed by $j = 1, \cdots, N$, each agent has an individual perceived law of motion consistent with the MSV solution of the model such as:

$$E^{SL}_{j,t}\widehat{y}_{t+1} = a_{1,j} + c_{1,j}\rho^g\widehat{g}_t + d_{1,j}\rho^u\widehat{u}_t \tag{21}$$

$$E^{SL}_{j,t}\widehat{\pi}_{t+1} = a_{2,j} + c_{2,j}\rho^g\widehat{g}_t + d_{2,j}\rho^u\widehat{u}_t \tag{22}$$

with the a, c and d coefficients are real numbers. Hence, every agent has a six coefficients strategy so that agent j is described by $\begin{bmatrix} a_{1,j} & c_{1,j} & d_{1,j} \\ a_{2,j} & c_{2,j} & d_{2,j} \end{bmatrix}$.

Contrary to Arifovic et al. (2012) and Arifovic et al. (2017) agents do not assess the fitness of their forecasts thought the history of their forecast errors, measured by the mean past absolute error. This process generate some time inconsistency when simulations were running for a long time. Thus, in this paper agents develops an auto-regressive fitness function. The auto-correlation coefficient is arbitrary equal to the real shocks' ones to ease the estimation process. Yet this hypothesis is consistent with the adaptive learning hypothesis as in Evans and Honkapohja (1999) where agent know the autocorrelation/persistence of exogenous.

$$F^y_{j,t} = \rho^g F^y_{j,t-1} - (1-\rho^g)(\widehat{y}_t - E^{SL}_{j,t-1}\widehat{y}_t)^2 \tag{23}$$

$$F^\pi_{j,t} = \rho^u F^\pi_{j,t-1} - (1-\rho^u)(\widehat{\pi}_t - E^{SL}_{j,t-1}\widehat{\pi}_t)^2 \tag{24}$$

Now, let us aggregate expectations over agents:

$$E^{SL}_t \widehat{\pi}_{t+1} = \frac{1}{N}\sum_{j=1}^{N} E^{SL}_{j,t}\widehat{\pi}_{t+1} \tag{25}$$

$$E^{SL}_t \widehat{y}^e_{t+1} = \frac{1}{N}\sum_{j=1}^{N} E^{SL}_{j,t}\widehat{y}_{t+1} \tag{26}$$

After clarifying the aggregation process, the five updating steps of the forecast rule can be described. In every period, agents' strategies follow sequentially the three standard genetic operators, namely: mutation, crossover (for the exploration part) and tournament (for the exploitation part).

Exploration: Mutation In each period, with an exogeneously fixed probability mu, each agent' coefficient changes as:

$$x_{t+1} = x_t + \iota_{x_t}\overline{X}\xi \tag{27}$$

With $x = a_1, a_2, c_1, c_2, d_1, d_2$ the value of the mutating coefficient, ι a random draw from a normal center distribution, ξ the mutation standard deviation and $\overline{X}$ the steady state value of the concerned variable (e.g output or inflation). Mutation is an operator that enables agents to find new solution or to discover better forecasts outside the agents' common knowledge. Mutation can be assimilated to a private signal, a noise within the solution space or a control error.

Exploitation: Tournament The next genetic operator is referred to as the tournament. This process is the selection force of the algorithm. All agents are randomly paired afresh and compare their fitness. The one with the lowest fitness copies the strategy and the fitness value of the other. Thus, poor-performing strategies tend to disappear in favor of better-performing ones.

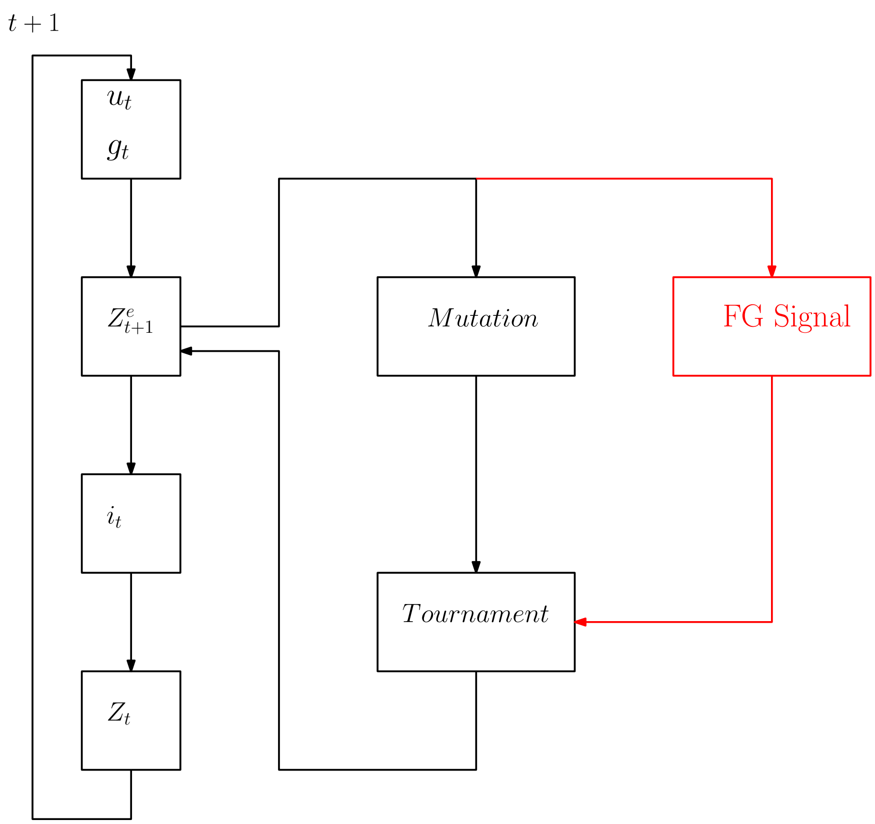

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Figure 3: Representation of the Social Learning Algorithm under Forward Guidance Treatment and credibility assessment (When the forward guidance treatment is not implemented every agent goes through the mutation branch)

Finally, in our model, liquidity traps arise as a result of a drop in expectations, not due to exogenous processes like in DSGE models with discount factor shocks following Markov processes (see e.g. Arifovic et al. (2017)).

Note that our algorithm is even simpler and more parsimonious than in Arifovic et al. (2012) or Arifovic et al. (2017), and the results below do not necessitate a learning phase nor do they depend on the initialization procedure.

4 Estimation Strategy

Here, mention, without stressing, especially regarding the calibration of the Phillips curve: 'the chosen calibration compensates the absence of frictions, especially consumption habits, that limits the ability of the simple three-equation model to fit the data. Adding such frictions would add state variables to the MSV solutions and considerably complicate the social learning process, without bringing further insights into the dynamics'.

Ergodicity of the model is a clear improvement upon previous literature.

We now proceed to estimate some of the model's key parameters, by matching empirical moment statistics with their theoretical counterpart from the model. We employ the Simulated Moments Method (SMM) to estimate the model's structural parameters as initially developed by McFadden (1989). The standard workhorse New Keynesian model - described in Equation 1 to 5 - can be expressed in the following compact form:

$$\mathbb{E}_t^* \left\{ f_\Theta \left(z_{t+1}, z_t, z_{t-1}, \varepsilon_t \right) \right\} = 0 \text{ for } * = \{RE, SL\} \tag{28}$$

where z_t is the set of endogenous variables, ε_t the set of iid innovations and $f_\Theta(\cdot)$ the model's equations using calibration Θ. We thus contrast the two possible ways of forming expectations $\mathbb{E}_t^i \{\cdot\}$ through standard rational expectations or through the social learning process. The SMM approach provides a regular basis to evaluate whether models in Equation 28 are able to replicate salient business cycle properties. In the following fit exercise, we originally consider the zero lower bound as an explicit objective to match along other standard business cycle moments. We employ the piecewise solution method developed by Guerrieri and Iacoviello (2015b) for the rational expectation model, while for the social learning one, we use an optimized version of algorithm of Arifovic et al. (2012) or Arifovic et al. (2017).[2]

We partition the parameters Θ into two sets: the first set contains mainly technology and preferences parameters which we calibrate following the litera-

[2] The mutation involved by the social learning may generate unstable dynamics when one (or a combination) of mutation(s) is too large. The solution usually adopted by Arifovic et al. (2012) is to draw a large number of parallel economies and select the median of them. However, the selection process of an average stable path is computationnally intensive and slows down the optimization exercise. We solve this issue by selecting the median, prior to the optimization, which allows to speed up the algorithm and makes the estimation as quick as for the rational expectation models. This selection process of the stable median prior the estimation is comparable to selecting the stable roots in the policy function of a rational expectation model.

ture. The second set $\theta \in \Theta$ contains parameters that we estimate by minimizing the distance between simulated and empirical moments.Θ contains parameters that we estimate by minimizing the distance between simulated and empirical moments.

4.1 Calibrated Parameters

Table 1 reports the set of calibrated parameters. To be comparable on a regular basis, we employ the same calibration between the rational expectation and the social learning algorithm. However, the social learning algorithm is more sophisticated and requires to fix an additional set parameters that shape the expectation formation and heterogeneity between agents.

For common parameters, our calibration is rather standard and mainly relies on the contribution of Woodford (2002). In particular, the consumption risk aversion coefficient is normalized to one, $\sigma = 1$, as well as the labor supply elasticity, $\varphi = 1$. We fix the Calvo parameter θ to 0.90, which is a relatively high value with respect to the canonical estimation of Smets and Wouters (2007a). However, the recent US experience, characterized by a combination of low inflation and nominal rate at the ZLB, has flatten the Phillips curve, which materializes in our model though a high value for the Calvo parameter (see Gourio et al. (2017) for a similar calibration). This allows to accurately portray this new weak correlation link between inflation and output since the financial crisis episode. Finally, regarding monetary policy reaction parameters, we employ the values estimated by Taylor (1993).

Regarding the calibration determining the social learning expectation formation process, it comprises 4 parameters. The first one, is the mutation standard deviation (ξ) which is not very significant. An higher ξ generates a bigger noise at the steady state and act as expectation amplificator without jeopardizing the stability of the model. We choose to fix this parameter at 0.1 which is consistent with previous findings of Arifovic et al. (2012, 2017). We depart from the latter by increasing the mutation standard deviation ξ up to 5% to adjust the expectation response consistently with empirical evidence. Mutation is proportional to the steady state of expected variables. We thus define the steady state of output $\overline{Y} = 1/3$ and inflation $\bar{\pi} = 0.005$. Since hours worked represents 1/3 of time per day for the US economy, output is given the same value via the production function. For inflation, we simply consider the average quarterly inflation rate for the US economy. A more crucial parameter is $0 < \mu < 1$ the learning parameter which stands for the probability to mutate and crossover. The higher μ is the higher mutation, crossover are happening. On the other side, the lower μ is the more consistent agents' expectations are if we initiate around the targeted steady state. Yet, the value of μ does not affect neither the stability nor the fundamental property of the model. Nonetheless a higher μ enables faster change in the perceived steady state level value and eases longer persistence at the steady state. Thus we set the calibration to $\mu = 0.25$ which is consistent with Arifovic et al. (2012) and higher than Arifovic et al. (2017). Finally, we set the agent population as in Arifovic et al. (2017) at $N = 300$ to

		Rational Expectations	Social Learning	Sources
σ	consumption risk	1	1	Woodford (2002)
φ	labor disutility curvature	1	1	Woodford (2002)
θ	calvo probability	0.90	0.90	Gourio et al. (2017)
ϕ^{π}	policy stance on inflation	1.50	1.50	Taylor (1993)
ϕ^{y}	policy stance on output	0.125	0.125	Taylor (1993)
$\bar{\pi}$	steady state inflation rate	$1.02^{0.25}$	$1.02^{0.25}$	US Data
Y	steady state output	1/3	1/3	Hansen (1985)
μ	mutation probability	×	0.25	Arifovic et al. (2012)
ξ	mutation standard deviation	×	0.05	Arifovic et al. (2012)
N	number of agents	×	300	Arifovic et al. (2012)
B	number of draws	×	100	Arifovic et al. (2012)

Table 1: Calibration of the model under alternative expectations

avoid like Arifovic et al. (2012) unjustified jump due to single mutation at a steady state when there is no perturbation.

4.2 Estimated Parameters

Let $m_T(x_t)$ be a $p \times 1$ vector of moments calculated using stationary and ergodic real data x_t of sample size T and $m_{s,\tau}\left(\hat{x}_t^{\theta}\right)$ the model-generated counterpart based on artificial series $\hat{x}_t^{\theta}$ of size τ generated using the set of parameters θ. To get an unconditional measure of simulated moments, we exploit asymptotic properties of Monte-Carlo methods by sampling s different sequences of shocks of size τ and compute the unconditional moment as an average from the s moments from each sequences.[3] Artificial series $\hat{x}_t^{\theta}$ are obtained from models in Equation 28 that are solved either with rational expectations or social learning. The SMM estimator is defined as:

$$\hat{\theta}_{SMM} = \arg\min_{\theta} \left[m_T(x_t) - m_{s,\tau}\left(\hat{x}_t^{\theta}\right)\right]' W \left[m_T(x_t) - m_{s,\tau}\left(\hat{x}_t^{\theta}\right)\right] \tag{29}$$

where $m_T(x_t) - m_{s,\tau}\left(\hat{x}_t^{\theta}\right)$ is the distance vector between the observed and the simulated moments that we seek to minimize, and W is the weighting matrix. The matrix product in Equation 29 provides the sum of the squares of the residuals between observed and matched moments. We solve Equation 29 using the *CMAES* optimization algorithm of **?**.[4]

[3] To ensure that each iteration of the optimization algorithm are performed on a regular basis, we randomly draw s different sequences of shocks of size τ at the initialization of the fit exercise, and keep them unchanged during the optimization. In this paper, we generate artificial series of size $\tau = 250$ and drop the first 50 draws. These artificial series are drawn $s = 40$ different times to approximate the unconditional moments used in the objective function.

[4] Contrary to other alternative algorithm, CMAES is able to deal with large scale optimization problem and provide an accurate measure of the hessian matrix even with bound restrictions for control variables in Equation 29.

To avoid identification issues, the number of parameters to be estimated is the same as the number of matched moments, so that each estimated parameter is directly mapped to an empirical moment. The mapping strategy reads as follows: standard deviations of shocks σ^g and σ^u capture the empirical volatility of output and inflation, while roots of the shocks capture the auto-correlation of observable variables. Finally, the discount factor is estimated to capture the zero lower bound probability. An increasing discount factor mechanically lowers the nominal rate through the Euler equation, and increases the likelihood of hitting the zero lower bound: the probability of hitting the zero lower bound $P\left[\tilde{r}_t \geq (1+\bar{\pi})/\beta - 1\right]$ is a function of β for a linear approximation to the policy function.[5]

To speed up the optimization exercise, we set bound restrictions for estimated parameters. These restrictions are deemed necessary with a model featuring the zero lower bound on the nominal interest, as the latter can be induce indeterminacy, in particular when the standard deviation of shocks is unbounded. We first estimate a NK-RE model without the ZLB, we find that the demand shock is ten times more volatile than for the supply shock. We thus set a bound restriction [0;1] for the demand shock, and [0;0.1] for the supply one. The persistence of shocks are imposed to be between 0.5 and 1. Finally, the discount factor support restriction [0.97;0.998] allows the quarterly nominal rate to lie in the interval [1.007;1.0361], which is fairly consistent with the US historical experience.

Estimated parameters are reported in Table 3 while matched moments are in Table 2. We first compare the matched moments to evaluate which model best replicates business cycle moments. Not surprisingly, the SL model outperforms the RE model with an objective function 35% lower. In particular, the SL model is better at capturing all of the moments except the autocorrelation of inflation. The latter is understated by the RE model and overstated by the SL model. Regarding estimated parameters, both models replicate business cycles moments using rather different sets of parameters. The RE model employs more persistent shocks combined with a larger demand shock and a smaller supply shock. The discount factor is also very different between models as the SL model is able to replicate the ZLB with a rather high nominal rate unlike the RE model.[6] This gap is mainly driven by endogenous stagnation traps that can arise within the SL model, when agents hold pessimistic expectations about future output and inflation. These pessimistic expectations allows the economy so stay for an extensive period of time at the ZLB (see 4).

[5]The ZLB binds when the following condition holds: $r_t = 1$. Thus, applying a first order approximation of this condition, $\bar{r}(1+\tilde{r}_t) = 1$, which can be rewritten as $\tilde{r}_t = \bar{r}^{-1} - 1 = (1+\bar{\pi})/\beta - 1$. The ZLB probability is thus approximated by: $P\left[r_t \geq 1\right] \simeq P\left[\tilde{r}_t \geq (1+\bar{\pi})/\beta - 1\right]$.

[6]The estimated SL model exhibits a nominal rate of 1.0193 the corresponding counterpart for the RE model is 1.0121.

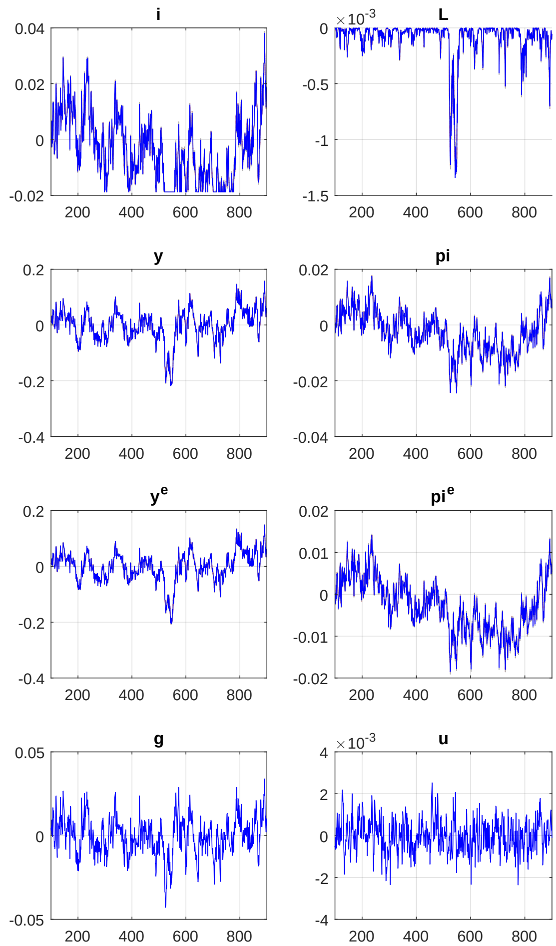

Figure 4: Sample of the model behavior on 100 draws after estimation

	Empirical Moments	Theoretical Moments Rational Expectations	Social Learning
$\sigma(\hat{y}_t)$ output std.	4.68	4.67	4.68
$\rho(\hat{y}_t, \hat{y}_{t-1})$ output autocorr.	0.98	0.83	0.86
$\sigma(\hat{\pi}_t)$ inflation std.	0.57	0.62	0.61
$\rho(\hat{\pi}_t, \hat{\pi}_{t-1})$ inflation autocorr.	0.86	0.83	0.93
$P(r_t = 1)$ ZLB probability	0.10	0.12	0.09
Objective function	$\times$	0.025	0.016

Table 2: Business cycle statistics comparison between estimated models

Estimated Parameters	Support [min;max]	Rational Expectations Mean	STD	Social Learning Mean	STD
σ^g - demand shock std	[0.000;1.000]	0.7137	6.47e-07	0.6200	1.01e-06
ρ^g - demand shock AR	[0.500;0.999]	0.9502	3.41e-07	0.8090	4.58e-07
σ^u - supply shock std	[0.000;0.100]	0.0186	2.16e-07	0.0613	2.39e-07
ρ^u - supply shock AR	[0.500;0.999]	0.8332	6.67e-07	0.6334	1.68e-07
β - discount factor	[0.970;0.998]	0.9928	6.05e-07	0.9860	8.04e-08

Notes: Confidence intervals are computed using the Hessian matrix.

Table 3: Estimated parameters using the simulated moment method

5 Stability under learning

We now examine the model stability properties through Monte-Carlo simulations. Stability under learning properties is a critical point in the bounded rationality literature in macroeconomics (Evans and Honkapohja, 1999, 2003; Bullard et al., 2002). Benhabib et al. (2001a), Hommes (2013), Evans et al. (2008) and Branch and McGough (2010) show determinacy and stability results in function of learning parameters, monetary rule coefficients and shocks magnitude in a similr class of models. Arifovic et al. (2012) and Arifovic et al. (2017) establish the stability under social learning in a linear NK model. In our model, the targeted steady state is stable under learning, as well as the stagnation state, but the deflationary one is unstable.

In order to generalize our conclusion, we simulate our model over a grid of initialization points of the $\begin{bmatrix} a_{1,i} \\ a_{2,i} \end{bmatrix}$ and display the dynamic in the solution space $(\widehat{\pi^e}, \widehat{y^e})$ (see 5). Those simulations show that the model converges to either the target - intersection between the black and green lines - or the stagnation -bottom left corner - equilibrium points (see 5). We can very much see the frontier between the two basins of attraction, given by the stable manifold associated to the deflationary state (which is a saddle under adaptive learning). Under rational expectation, above the manifold the Taylor condition -red line- is respected and all expectation pairs converge to the target, below, the conditions

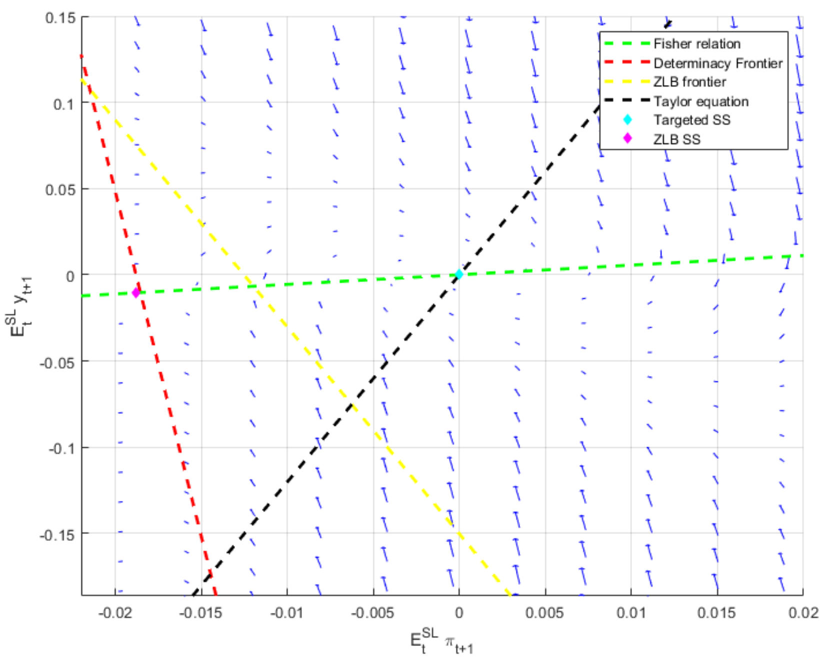

Figure 5: Learning Stability of the social learning

are violated, and we obtain convergence to the stagnation state(see figures 4).[7] Those results are in line with previous stability results under adaptive learning, and determinacy results under rational expectations . Yet, under this initialization setup where there is no real shocks we can see that our model is able to violate for some time Taylor condition - on the top left zone of the diagram -and get back to equilibrium. According to our simulation this result is more a consequence of diagram initialization setup because under real shock the model's dynamics does not allow this situation - e.g strong deflationary expansion.

6 Results

One of the most topical issues regarding RE NK models with multiple steady state and non-linearities is how to compute regime switches and the corresponding ZLB duration. Iacoviello and Luca (2014) develop a way to achieve realistic values by computing it through an algorithm that estimates rather than analytically computing duration of constraint binding within the policy function. One of the purposes of the social learning model is to demonstrate how coordination issues and selection mechanisms can increase the persistence of shock. To assess the efficiency of policy we developed a welfare policy function in this fashion :

$$L_t = -(\pi_t - E_{t-1}\pi_t)^2 - \lambda y^2 \tag{30}$$

with $\lambda = \kappa/\theta$ such as in Woodford (2002).

6.1 Impulse response functions

After observing the stability properties of the model in the simulated economies, we investigate the dynamic properties and responses of the model to natural rate and expectation shocks relative to the rational expectation benchmark.

One positive demand real shock shows that our model responds correctly to this kind of impulse. One striking result is that if aggregate expectations follow the shock, the *a* constant is positively correlated as if an exogenous shock changes steady states values or structural values of the economy. That phenomenon explains the higher shock persistence of this model (see figures 6.1 and 6.1). The intuition behind those phenomena is that agent struggle to identify the underlying disturbance and could perceived an exogenous shock as a structural change -i.e change in the SS value.

One important feature of this model is that we can shock directly model expectations in the form of higher or lower inflation and output forecasts. This means that "news" can trigger by themselves real effect on the economy. Lower output expectations generate depressive episodes and higher inflation expectations can push output gap up. Yet, because of identification issue during the estimation part those shocks are quiet challenging to use and we refrained the use of those shocks only to the forward-guidance part.

[7] In every IRF, blue line is the median of Monte-Carlo results, grey lines are the upper and lowers quartiles and green, yellow and red lines the three steady states.

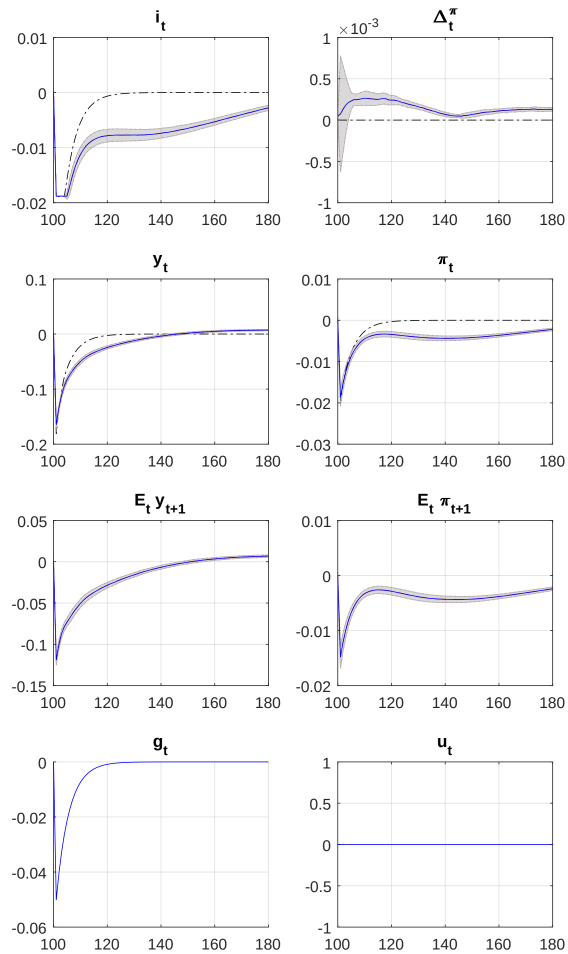

Figure 6: IRF of the model under a demand shock $\epsilon_1^g = -0.05$ (blue line SL, grey band 95% confidence interval, black line RE) 1/2

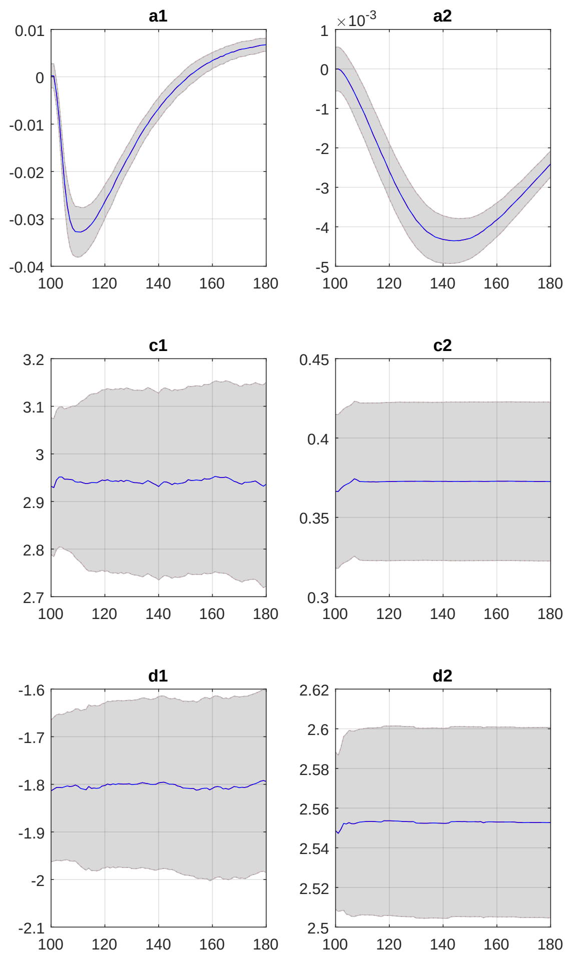

Figure 7: IRF of the model under a demand shock $\epsilon_1^g = -0.05$ (blue line SL, grey band 95% confidence interval, black line RE) 2/2

In our model exogenous shocks -i.e real shocks- can also lead to self-sustained recession paths and deflationary spirals. When shocks are large enough, coordination failures, i.e. the failure of agents to coordinate their expectations on the socially desirable targeted steady state, appear when the Taylor condition is violated for to long. The model deviate from the targeted stable equilibrium and falls into the liquidity trap for a very long term. In fact it seems that inflation expectations become unanchored. In fact, coordination failure pushes the model into a longer depression. Indeed, if deflation accelerates for a limited number of period the social learning will start discarding positive or zero a^1 and a^2 coefficients and even after the end of the shocks persistence, the model could hold deflation. Thus, contrary to the RE homogeneous agent model, where Taylor condition should be broken to generate indeterminacy, social learning leads to lower long term stability and more flexible short term path.

6.2 Forward Guidance treatment

Look at price dispersion in the NKPC in Woodford, get somehow appear the variance of expectations between agents (cross-sectional) and motivate the policy scenarios by the need of coordinating inflation expectations to avoid heterogeneous expectations, and the associated price dispersion loss.

Contrary to rational expectation DSGE where an expected variable is equal to the variable at the next period plus model respons to the autocorrelation of the stochastic perturbation, our model separates those variables. Thus, we can submit those expectations to policy treatment. Our explicit modeling of the expectation formation process allows us to separate the model reactions from the expectations reaction in face of policy shocks. The question is thus how policy treatments can steer back expectations to the targeted steady state?

6.2.1 Signaling the MSV solution

Here, mention somewhere that we model a continuous injection that is costless, but in reality of course, not a panacea, as communicating a solution that is not credible may be costly in terms of credibility for the central bank.

Our second treatment can be described as delphic forward guidance in the sense that the central bank signals the MSV RE solution to the agents (see figure 2), see also Marzioni (2014) and Goy et al. (2016). The CB forecast is thus describe as follow :

$$Ez^e_{t+1,CB} = a^{CB} + c^{CB}\rho^g\widehat{g}_t + d^{CB}\rho^u\widehat{u}_t \tag{31}$$

with if $\phi^y(\overline{a}_1 + \overline{c}_1\rho^g\widehat{g}_t + \overline{d}_1\rho^u\widehat{u}_t) + \phi^\pi(\overline{a}_2 + \overline{c}_2\rho^g\widehat{g}_t + \overline{d}_2\rho^u\widehat{u}_t) > -\overline{r}$:

$$\begin{cases} a_{CB} = \overline{a} \\ c_{CB} = \overline{c} \\ d_{CB} = \overline{d} \end{cases} \tag{32}$$

if $\phi^y(\overline{a}_1 + \overline{c}_1\rho^g\widehat{g}_t + \overline{d}_1\rho^u\widehat{u}_t) + \phi^\pi(\overline{a}_2 + \overline{c}_2\rho^g\widehat{g}_t + \overline{d}_2\rho^u\widehat{u}_t) < -\overline{r}$:

$$\begin{cases} a_{CB} = a^{ZLB} \\ c_{CB} = c^{ZLB} \\ d_{CB} = d^{ZLB} \end{cases} \tag{33}$$

and if $\overline{a}_2 + \overline{c}_2\rho^g\widehat{g}_t + \overline{d}_2\rho^u\widehat{u}_t < -\overline{r}$:

$$\begin{cases} a_{CB} = \underline{a} \\ c_{CB} = \underline{c} \\ d_{CB} = \underline{d} \end{cases} \tag{34}$$

We assume that there is a fixed proportion $0 < \psi < 1$ of receivers among our agents. Those agents do not proceed to mutation but proceed through the tournament so that the social learning algorithm endogenously spreads out or dismisses those forecasts, depending on their relatives performances (see 3.3). This Forward Guidance treatment can be envisioned as a public signal send to some agent. Either agent are listening the CB news or the just have an intuition on the future state of the economy.

Interestingly, this treatment is very efficient in steering faster agents' beliefs towards the targeted state, even with a quite small proportion of initial receivers (e.g 5%) (see figures 6.2.1 and 6.2.1. Yet we discovered a trade-off between anchorage and Welfare and Dispersion of inflation forecast. This is happening because forward guidance can be overly pessimistic at the resolution of the shock. Moreover, the CB forecast could be way off the average forecast thus creating coordination issues. Thus, our forward guidance treatment can be suitable on long run for macro-economic stabilization but can generate signification welfare losses (see 4, 5 ahd 6) Yet when the shocks are too important (i.e beyond the Taylor conditions) or when the number of believers is significant this treatment tend to increase the magnitude of the shocks in the short run and the instability of the model in the long run.

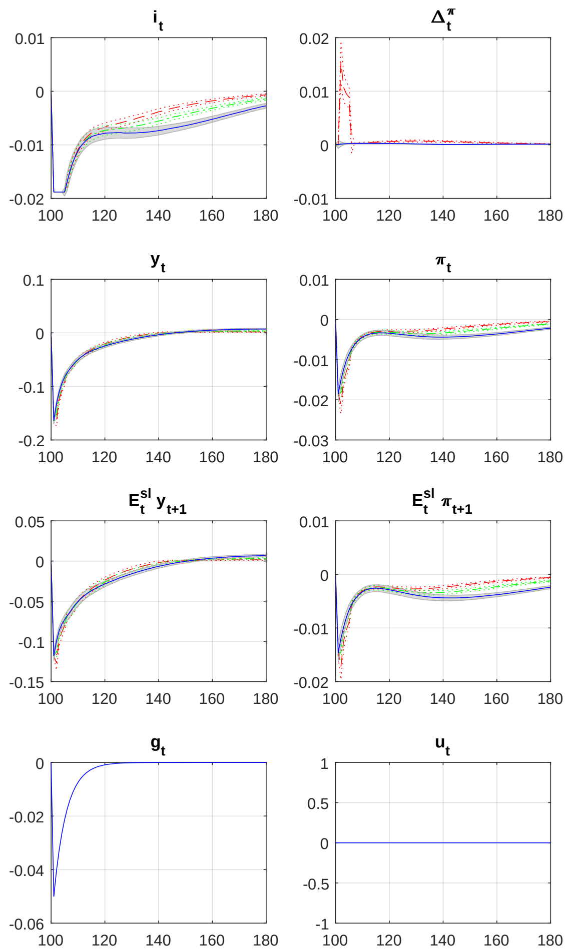

Figure 8: IRF of the model under a demand shock $\epsilon_1^g = -0.05$ and forward guidance treatment (blue line SL, grey band 95% confidence interval,red line $\psi = 0.1$, green line $\psi = 0.05$) 1/2

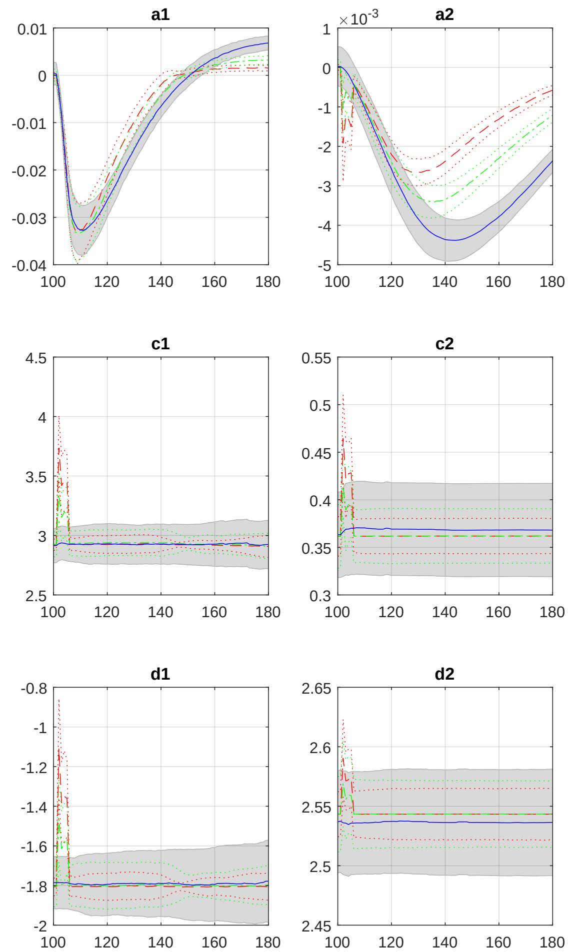

Figure 9: IRF of the model under a demand shock $\epsilon_1^g = -0.05$ and forward guidance treatment (blue line SL, grey band 95% confidence interval,red line $\psi = 0.1$, green line $\psi = 0.05$) 2/2

$\psi \diagdown \hat{g}_1$	$\hat{g}_1 = -.005$	$\hat{g}_1 = -.01$	$\hat{g}_1 = -.015$	$\hat{g}_1 = -.02$	$\hat{g}_1 = -.025$	$\hat{g}_1 = -.03$	$\hat{g}_1 = -.035$	$\hat{g}_1 = -.04$	$\hat{g}_1 = -.045$	$\hat{g}_1 = -.05$	$\hat{g}_1 = -.055$	$\hat{g}_1 = -.06$
$\psi = .00$	.576	.6	.691	.761	.824	.881	0.93	.968	1	1.03	1.06	1.07
$\psi = .01$	.626	.793*	.983**	1.14***	1.26***	1.6***	1.71***	1.94***	2.04***	2.27***	2.35***	2.56***
$\psi = .02$	.657	.876**	1.09***	1.28***	1.42***	1.84***	2.13***	2.5***	2.4***	2.89***	3***	3.3***
$\psi = .03$	.676*	.933***	1.17***	1.35***	1.51***	2.02***	2.38***	2.77***	2.58***	3.29***	3.42***	3.78***
$\psi = .04$	.686*	.963***	1.21***	1.40***	1.56***	2.11***	2.53***	2.96***	2.91***	3.56***	3.7***	4.08***
$\psi = .05$	.691*	.978***	1.23***	1.42***	1.57***	2.17***	2.64***	3.12***	3.12***	3.76***	3.95***	4.40***
$\psi = .06$	.692*	.982***	1.22***	1.41***	1.56***	2.26***	2.75***	3.26***	3.29***	3.92***	4.08***	4.56***
$\psi = .07$	.69*	.981***	1.21***	1.41***	1.55***	2.26***	2.81***	3.36***	3.43***	4.05***	4.23***	4.74***
$\psi = .08$	.677*	.971***	1.18***	1.39***	1.52***	2.27***	2.83***	3.37***	3.52***	4.12***	4.29***	4.84***
$\psi = .9$	.669	.959***	1.16***	1.37***	1.48***	2.26***	2.84***	3.41***	3.53***	4.17***	4.35***	4.94***
$\psi = .10$	.656	.941***	1.14***	1.34***	1.46***	2.39***	2.88***	3.49***	3.58***	4.29***	4.48***	5.08***
$\psi = .11$	.643	.924***	1.11***	1.31***	1.42***	2.37***	2.88***	3.52***	3.66***	4.39***	4.58***	5.23***
$\psi = .12$	.631	.906***	1.09***	1.28***	1.38***	2.36***	2.89***	3.55***	3.73***	4.40***	4.6***	5.28***
$\psi = .13$	.617	.88**	1.05***	1.25***	1.35***	2.34***	2.87***	3.55***	3.74***	4.45***	4.65***	5.35***
$\psi = .14$	.606	.861**	1.02***	1.22***	1.31***	2.33***	2.88***	3.59***	3.77***	4.50***	4.69***	5.43***
$\psi = .15$	.592	.84**	.992***	1.18***	1.27***	2.33***	2.86***	3.64***	3.84***	4.51***	4.72***	5.47***
$\psi = .16$	.58	.82**	.962**	1.15***	1.23***	2.29***	2.83***	3.70***	3.89***	4.54***	4.74***	5.54***
$\psi = .17$	.57	.793*	.938**	1.12***	1.19***	2.25***	2.95***	3.71***	3.91***	4.57***	4.77***	5.69***
$\psi = .18$	.558	.75	.906*	1.07**	1.16**	2.22***	2.92***	3.70***	3.91***	4.71***	4.94***	5.74***
$\psi = .19$	.551	.75	.892*	1.03**	1.13**	2.20***	2.92***	3.70***	3.90***	4.72***	4.96***	5.79***
$\psi = .2$	.546	.73	.882	1.01**	1.11**	2.19***	2.92***	3.71***	3.92***	4.76***	4.99***	5.84***

Table 4: Average price dispersion on 80 periods under different policy experiment scale $\Delta 10^4$

$\psi \diagdown \hat{g}_1$	$\hat{g}_1 = -.005$	$\hat{g}_1 = -.01$	$\hat{g}_1 = -.015$	$\hat{g}_1 = -.02$	$\hat{g}_1 = -.025$	$\hat{g}_1 = -.03$	$\hat{g}_1 = -.035$	$\hat{g}_1 = -.04$	$\hat{g}_1 = -.045$	$\hat{g}_1 = -.05$	$\hat{g}_1 = -.055$	$\hat{g}_1 = -.06$
$\psi = 0$	78	100	101	102	103	104	105	106	107	108	109	109
$\psi = .01$	72	94	99	101	101	103	104	105	105	106	107	107
$\psi = .02$	63	90*	95	98	100	100	102	103	104	105	105*	106
$\psi = .03$	51	84***	92**	95*	97*	98**	100**	101**	102**	102***	103***	104**
$\psi = .04$	47	80***	87***	91***	93***	96***	97***	99***	99***	101***	101***	102***
$\psi = .05$	40	74***	83***	87***	90***	93***	95***	97***	97***	98***	99***	100***
$\psi = .06$	40	69***	79***	83***	86***	90***	92***	93***	95***	95***	96***	97***
$\psi = .07$	32	64***	74***	79***	83***	86***	89***	91***	91***	93***	93***	94***
$\psi = .08$	32	59***	70***	75***	79***	83***	85***	87***	88***	90***	90***	91***
$\psi = .09$	26	54***	65***	71***	75***	79***	82***	84***	84***	86***	87***	88***
$\psi = .1$	22	49***	61***	67***	70***	75***	78***	80***	81***	83***	83***	85***
$\psi = .11$	18	45***	57***	63***	67***	72***	75***	76***	78***	79***	80***	82***
$\psi = .12$	16	42***	53***	59***	63***	68***	71***	73***	74***	76***	77***	78***
$\psi = .13$	15	38***	49***	56***	59***	64***	67***	70***	71***	72***	73***	75***
$\psi = .14$	14	35***	45***	52***	57***	61***	64***	66***	67***	69***	70***	72***
$\psi = .15$	13	33***	43***	49***	53***	58***	61***	63***	64***	66***	67***	68***
$\psi = .16$	12	31***	40***	46***	50***	55***	57***	59***	61***	63***	64***	65***
$\psi = .17$	12	28***	37***	43***	47***	52***	54***	57***	58***	60***	61***	62***
$\psi = .18$	12	27***	35***	40***	44***	49***	52***	54***	55***	57***	58***	59***
$\psi = .19$	12	25***	33***	38***	42***	46***	49***	51***	52***	54***	55***	57***
$\psi = .2$	12	23***	31***	37***	40***	43***	46***	48***	49***	51***	52***	54***

Table 5: Time for expectations to reach back the inflation target under different policy experiment

$\psi \diagdown \hat{g}_1$	$\hat{g}_1 = -.005$	$\hat{g}_1 = -.01$	$\hat{g}_1 = -.015$	$\hat{g}_1 = -.02$	$\hat{g}_1 = -.025$	$\hat{g}_1 = -.03$	$\hat{g}_1 = -.035$	$\hat{g}_1 = -.04$	$\hat{g}_1 = -.045$	$\hat{g}_1 = -.05$	$\hat{g}_1 = -.055$	$\hat{g}_1 = -.06$
$\psi = 0$	-.028	-.116	-.267	-.478	-.749	-1.1	-1.52	-2	-2.55	-3.16	-3.84	-4.57
$\psi = .01$	-.027	-.1141	-.263	-.472	-.739	-1.1	-1.52	-2.01	-2.56	-3.18	-3.86	-4.61
$\psi = .02$	-.027	-.112	-.256	-.464	-.727	-1.1	-1.52	-2.02	-2.57	-3.2	-3.87	-4.63
$\psi = .03$	-.026	-.11	-.255	-.475	-.718	-1.1	-1.53	-2.03	-2.58	-3.21	-3.9	-4.66
$\psi = .04$	-.026	-.109	-.253	-.454	-.712	-1.11	-1.54	-2.04	-2.60	-3.24	-3.96	-4.70
$\psi = .05$	-.026	-.108	-.25	-.45	-.705	-1.13	-1.56	-2.07	-2.63	-3.27	-4.02	-4.75
$\psi = .06$	-.026	-.107	-.248	-.446	-.7	-1.15	-1.58	-2.11	-2.66	-3.33	-4.06	-4.82
$\psi = .07$	-.026	-.106	-.247	-.444	-.697	-1.17	-1.60	-2.13	-2.7	-3.36	-4.12	-4.87
$\psi = .08$	-.026	-.105	-.245	-.441	-.693	-1.19	-1.62	-2.16	-2.73	-3.42	-4.15	-4.91
$\psi = .09$	-.025	-.105	-.243	-.438	-.688	-1.2	-1.64	-2.18	-2.75	-3.44	-4.23	-4.93
$\psi = .1$	-.025	-.105	-.242	-.436	-.684	-1.24	-1.68	-2.23	-2.82	-3.51	-4.25	-4.97
$\psi = .11$	-.025	-.104	-.241	-.434	-.682	-1.28*	-1.71	-2.27	-2.87	-3.57	-4.28	-5
$\psi = .12$	-.025	-.104	-.24	-.432	-.679	-1.31*	-1.74	-2.3*	-2.9	-3.62*	-4.32*	-5.03
$\psi = .13$	-.025	-.103	-.239	-.431	-.676	-1.34**	-1.178*	-2.36**	-2.96*	-3.69*	-4.35*	-5.07
$\psi = .14$	-.025	-.103	-.238	-.43	-.673	-1.39***	-1.83**	-2.41**	-3.03**	-3.74**	-4.37*	-5.1
$\psi = .15$	-.025	-.103	-.238	-.426	-.67	-1.41***	-1.87**	-2.46**	-3.07**	-3.76**	-4.41*	-5.13
$\psi = .16$	-.025	-.102	-.237	-.424	-.668	-1.45***	-1.9***	-2.51***	-3.14**	-3.79**	-4.44*	-5.18
$\psi = .17$	-.025	-.102	-.236	-.422	-.6643	-1.49***	-1.96***	-2.56***	-3.19***	-3.82**	-4.46*	-5.21*
$\psi = .18$	-.025	-.102	-.234	-.421	-.662	-1.56***	-2.03**	-2.64***	-3.24***	-3.85**	-4.47*	-5.25*
$\psi = .19$	-.025	-.102	-.233	-.42	-.66	-1.62***	-2.09***	-2.71***	-3.27***	-3.87**	-4.49*	-5.29
$\psi = .2$	-.025	-.102	-.232	-.419	-.658	-1.69***	-2.15***	-2.78***	-3.31***	-3.91***	-4.55**	-5.35**

Table 6: Average welfare loss on 80 periods under different policy experiment Ł10^{-6}

6.2.2 Signaling a policy commitment: the "whatever it takes" experiment

Following previous theoretical and empirical works by Black (1995), Kulish et al. (2014) and Bauer and Rudebusch (2016), forward guidance at the ZLB could be considered as a way to decrease the interest rate under 0%. In fact, forward guidance can be envisioned as long term commitment to flatten and even render negative the rate curve through the risk and money supply channels. This enables the lending conditions to be eased and the de facto riskless rate to go below the ZLB. This *de facto* rate has been conceptualized as the "shadow rate". For instance, Kulish et al. (2014) and Bauer and Rudebusch (2016), thanks to a Bayesian DSGE model Ãă la Smets and Wouters (2007b) or a Bayesian VAR model, estimate that the forward guidance of Federal Reserve has generated a "shadow" interest rate of about -3% between 2009 to 2013. In this part, we develop a shadow interest setup without modifying the actual interest determination and modifying the ZLB constraint. We instead break the perfect foresight equality of the MSV central bank signal that we can describe as the shadow rate of the monetary policy. Hence, the shadow/perceived interest rate is now different from the actual rate.

The forward-guidance system is based on the solution signaled by the central bank and can be described in this fashion :

$$Ez^e_{t+1,CB} = a^{WIT} + c^{WIT}\rho^g\widehat{g}_t + d^{WIT}\rho^u\widehat{u}_t \tag{35}$$

with:

$$\begin{cases} a^{WIT} = \overline{a} \\ c^{WIT} = \overline{c} \\ d^{WIT} = \overline{d} \end{cases} \tag{36}$$

Without changing any fundamentals of the model - namely the ALM, we implement an Odysean forward guidance treatment. Indeed, we delete the ZLB constraint in the MSV solution signaled by the central bank. The intuition is that central bankers signal that the will do "whatever it takes" to fulfill the Taylor principle. In a sense, this type of forward guidance is a pure Odysean commitment from the central bankers that they will solve any problem regarding the ZLB with quantitative easing, maturity transformation, assets swap etc. to flatten the yield curve. In theory, if the central bank is perfectly credible, our model would be totally stable whatever shock will happen. The treatment is efficient and manage to anchor expectations efficiently (see figures 6.2.2 and 6.2.2). On the short run, the treatment under credibility assessment does not yield to important perturbation and thus forecasts dispersion and welfare losses. Yet, on the long run this "whatever it takes" policy increases the volatility of the model.

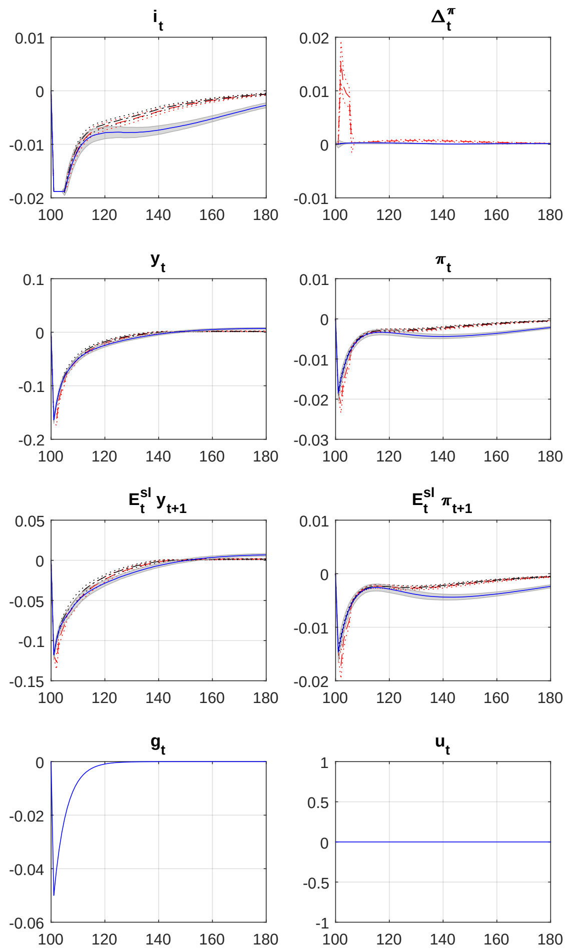

Figure 10: IRF of the model under a demand shock $\epsilon_1^g = -0.05$ under different treatments of odyssean forward guidance (blue line SL, grey band 95% confidence interval,red line $\psi = 0.1$ Delphic FG, green line Odysean FG $\psi = 0.1$) 1/2

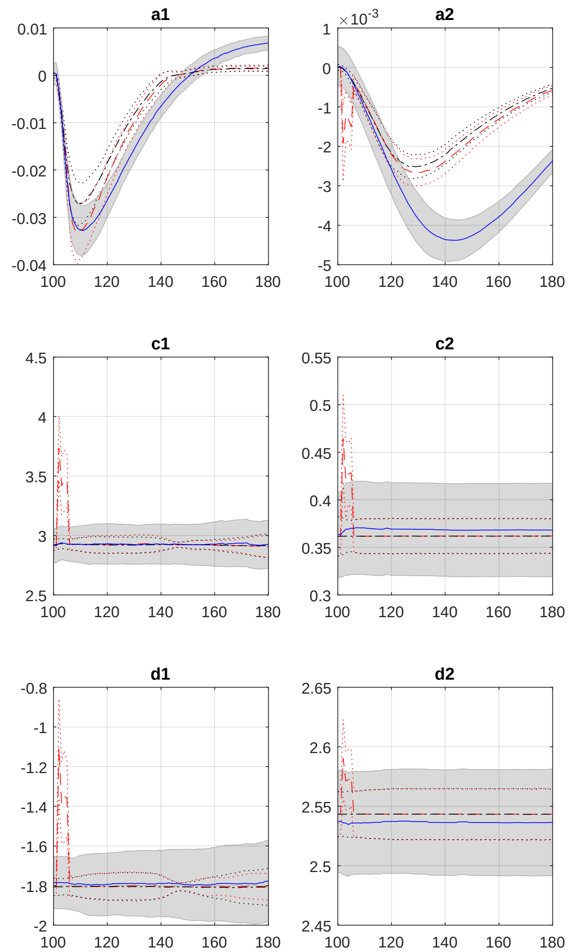

Figure 11: IRF of the model under a demand shock $\epsilon_1^g = -0.05$ under different treatments of odyssean forward guidance blue line SL, grey band 95% confidence interval,red line $\psi = 0.1$ Delphic FG, green line Odysean FG $\psi = 0.1$) 2/2

7 Conclusion

We introduce heterogeneous expectations through social learning in a NK model with multiple steady states. Heterogeneous expectations potentially undermine the effects of forward-guidance by the coordination dynamics implied by heterogeneity. Our model exhibits persistence at the ZLB and could fall into a liquidity trap state, which is a feature that standard RE NK models cannot easily produce.

Our model exhibits two interesting properties – namely non stationary and bounded rationality – that make forward-guidance potentially relevant. We experiment with so-called Delphic forward guidance in the form of central bank's signals about the MSV solution. Under full credibility, this policy treatment is very efficient at anchoring the expectations back on the socially efficient target and not efficient in term of welfare loss and forecast dispersion. We further implement so-called Odysean forward guidance in the form of an optimal commitment from the central bank to fulfill the Taylor conditions. This policy experiment enables us to lift the welfare loss and forecast dispersion constrain.

Coordination of heterogeneous expectations is a crucial issue for forward guidance and expectation management by central bank, and this feature is well captured by the model that we have developed in this paper thanks to the beauty contest property of forecasts. We also account for the well-known trade-off between credibility and transparency, that is even more salient in crisis times, once the economy evolves or is at risk of evolving far away from the central bank's objectives: communication may steer expectations towards the desirable state, but may also lock them into the stagnation state if the agents do not judge the central bank's information credible and do not incorporate it into their forecasts. Further attempts from the central bank to influence expectations through communication may turn out pointless if the credibility of their announcement has been lost. In our model, it seems that even communicating to agent the solution is already a powerful policy tool.

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